Mental Math Review Exercises – Multiplication, Part Two

This worksheet contains another fifty multiplication exercises in five sets of 10 problems. As with the last worksheet, there are suggestions to help you get faster. You can complete this worksheet several times before taking the AP exam; it is unlikely that you will memorize every answer, so each time you complete the worksheet gives you more practice with mental math.

Remember, it can be very helpful to rewrite the numbers on paper to help you "see" the calculation more clearly. It's ok to us a pencil and paper to do math; only calculators are forbidden.

<u>Famous fractions</u>

You must recognize a fraction when you see it in decimal form. For example, you probably instantly recognize 0.5 and 0.25 as the fractions ½ and ¼.

Here are the fractions that are most helpful to remember. Memorize them.

$\frac{1}{2} = 0.5$ $\frac{1}{4} = 0.25$ $\frac{3}{4} = 0.75$

$\frac{1}{5} = 0.2$ $\frac{2}{5} = 0.4$ $\frac{3}{5} = 0.6$ $\frac{4}{5} = 0.8$

$\frac{1}{3} = 0.\bar{3}$ $\frac{2}{3} = 0.\bar{6}$

The last two fractions, $\frac{1}{3}$ and $\frac{2}{3}$, are not often used for the types of calculations found in multiple choice questions.

More commonly, you will find multiples of $\frac{1}{8}$ used on the AP exam. For that reason, it is good to memorize the following fractions, too.

$\frac{1}{8} = 0.125$ $\frac{3}{8} = 0.325$ $\frac{5}{8} = 0.625$ $\frac{7}{8} = 0.875$

Sometimes, you might need to use $\frac{1}{16}$ in your calculations, so it's good to remember that:

$\frac{1}{16} = 0.0625$

These fractions also give you a chance to review half-life calculations. The half-life of a radioactive isotope is the amount of time required for half of the radioactive atoms to decay. The half-life of a chemical reaction is the amount of time needed for half of the reactants to disappear as they turn into a product. After one half-life, $\frac{1}{2}$ of the original atoms remain. After two half-lives, only $\frac{1}{4}$ remain. After three half-lives, $\frac{1}{8}$ of the original atoms remain. After only four half-lives, only $\frac{1}{16}$ remain. In other words, after four half-lives, only 6.25% of the original atoms are still present, while 93.75% have disappeared.

Mental Math with Famous Fractions

When using mental math, convert decimal numbers to fractions. Then, use the fractions to find the answer to the calculation. Finally, convert the answer back to a decimal number.

Example #1.

$$0.75 \times 0.5 = ?$$

Convert to fractions and multiply to get an answer.

$$\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$$

You should recognize that the final answer is 0.375.

Example #2.

$$0.375 \times 0.8 = ?$$

Converting the numbers to fractions allows you to simplify the final answer quickly.

$$\frac{3}{8} \times \frac{4}{5} = \frac{12}{40} = \frac{3}{10} = 0.3$$

Example #2 is a typical chemistry calculation. Here it is, rewritten with chemical symbols.

Question: How many moles of Na^{1+} are present in 375 mL of 0.80 M NaCl ?
Answer: 0.30 moles Na^{1+}.

(continued on the next page)

Distribution

Don't forget to recognize distribution when it can help simplify multiplication.
The fraction $\frac{1}{40}$ may look confusing, but it helps to rewrite it as $\frac{1}{4} \times \frac{1}{10}$.
In decimals, one fortieth is just 0.25 x 0.10 = 0.025.

Examine these two examples to see how distribution can help.

$$\frac{1}{8} \times \frac{1}{5} = \frac{1}{40} = \frac{1}{4} \times \frac{1}{10} = 0.25 \times \frac{1}{10} = 0.025$$

and

$$\frac{3}{8} \times \frac{2}{5} = \frac{6}{40} = \frac{6}{4} \times \frac{1}{10} = \frac{3}{2} \times \frac{1}{10} = 1.5 \times \frac{1}{10} = 0.15$$

Set 1 – Use the skills shown above to perform these calculations *without a calculator*. Convert each number to a fraction, multiply the fractions, then simplify the final answer and change it back into a decimal number.

1. 0.125 x 0.800 =
2. 0.625 x 0.800 =
3. 0.200 x 0.125 =
4. 0.125 x 0.400 =
5. 0.375 x 0.200 =
6. 0.500 x 0.125 =
7. 0.625 x 0.400 =
8. 0.400 x 0.875 =
9. 0.600 x 0.125 =
10. 0.875 x 0.800 =

(continued on the next page)

Chemistry Calculations: Mass of a Solute

The types of calculations shown above can appear in chemistry calculations. In the next set of problems, you will calculate the mass of solute needed to make a given volume of solution. As you know, the calculation requires two steps:

- Use n = C x V to calculate the number of moles of solute needed,
- Multiply by the molar mass to determine the number of grams of solute needed.

Three compounds show up repeatedly in such calculations on the AP exam. These compounds are used because of their molar mass is a number that is easily manipulate with mental math.

- NaOH, 40.0 g mol^{-1}
- glucose, $C_6H_{12}O_6$, 180 g mol^{-1}
- copper (II) sulfate pentahydrate, $CuSO_4{\cdot}5H_2O$, 250.0 g mol^{-1}

Set 2 – Use the molar masses shown above to perform these calculations *without a calculator*.

1. What mass of NaOH is needed to make 250 mL of 0.5 M solution?
2. What mass of $CuSO_4{\cdot}5H_2O$ is needed to make 100 mL of 0.2 M solution?
3. What mass of glucose is needed to prepare 125 mL of 0.8 M solution?
4. What mass of NaOH is needed for 250 mL of 0.4 M solution?
5. What mass of glucose is needed to prepare 125 mL of 0.2 M solution?
6. What mass of $CuSO_4{\cdot}5H_2O$ is needed to make 800 mL of 0.375 M solution?
7. What mass of glucose is needed for 400 mL of 0.125 M solution?
8. What mass of NaOH is needed to prepare 500 mL of 0.25 M solution?
9. What mass of $CuSO_4{\cdot}5H_2O$ is needed to make 125 mL of 0.20 M solution?
10. What mass of NaOH is needed for 125 mL of 0.60 M solution?

Chemistry Calculations: Molarity of a Solution

In this section, you will practice calculating the concentration of a solution when given mass of a solute and volume of the solution. As you know, two steps are required for this calculation:

- convert mass to moles; divide mass by molar mass
- calculate the concentration; divide moles of solute by volume of solution

Here are three comments about the process:

1. The mass of the solute is usually given in milligrams (mg), not grams. The molar mass, however, will be listed in grams. You must convert milligrams to grams. Since 1000 mg = 1 g, divide the mass in grams by 1000 to convert to mg. This is like the conversion from mL to L. Hint: numbers less than one are sometimes easier to manage if you write them in scientific notation. When doing mental math, 0.001 is not as easy to manage as 1.0×10^{-3}. You can also write 0.001 as $\frac{1}{1000}$
2. The most common volumes used in chemistry are between 100 mL and 500 mL. These have to be converted to liters, 0.1 L to 0.5 L, before they can be used to calculate molarity. For mental math, it is better to change these decimal numbers to fractions. When you divide by a fraction, you multiply the numerator by the reciprocal of the denominator. Therefore, dividing by 0.1 is the same as multiplying by 10. (See example below)
3. Occasionally, the given volumes will be smaller than 100 mL, from 20 mL to 75 mL. After converting these numbers to liters, you will have 0.020 L and 0.075 L. Again, for mental math, it is useful to change these numbers to fractions when you divide. Using distribution, we can find the fractions that are equal these decimal numbers.

$$0.02 = 0.1 \times 0.2 = \frac{1}{10} \times \frac{1}{5} = \frac{1}{50}$$

$$0.075 = 0.1 \times 0.75 = \frac{1}{10} \times \frac{3}{4} = \frac{3}{40}$$

Example 1

Calculate the molarity of a solution that contains 4.0 g of NaOH dissolved in 500 mL of solution.

4.0 g = 0.10 mol

500 mL = 0.5 L

$$C = \frac{n}{V} = \frac{0.10 mol}{0.5 L} = \frac{\frac{1}{10}}{\frac{1}{2}} = \frac{1}{10} \times \frac{2}{1} = \frac{2}{10} = 0.20 M$$

Notice that dividing by 0.5 is the same operation as multiplying by 2.

Example 2

Calculate the molarity of a solution that contains 120 mg of NaOH in 75 mL of solution.
First, convert mass to moles:

$$n = \frac{m}{MM} = \frac{120mg}{40.0g/mol} = \frac{120\times10^{-3}g}{40.0g/mol} = \frac{120}{40.0}\times10^{-3}mol = 3.0\times10^{-3}mol$$

Next, divide moles of solute by volume of solution.

$$C = \frac{n}{V} = \frac{3.0\times10^{-3}mol}{0.075L} = \frac{3.0\times10^{-3}g}{\frac{3}{40}L} = 3.0\times10^{-3}\times\frac{40}{3}M = \frac{3.0\times40}{3}\times10^{-3}M = 40\times10^{-3}M = 0.040M$$

Recall that 0.075 = $\frac{3}{40}$ as shown on the previous page.

Set 3 - Use the molar masses shown in the list below to perform these calculations *without a calculator*.

Three compounds show up repeatedly in such calculations on the AP exam. These compounds are used because of their molar mass is a number that is easily manipulate with mental math.

- NaOH, 40.0 g mol^{-1}
- glucose, $C_6H_{12}O_6$, 180 g mol^{-1}
- copper (II) sulfate pentahydrate, $CuSO_4{\cdot}5H_2O$, 250.0 g mol^{-1}

1. A student dissolves 4.0 g of NaOH in enough H_2O to make 500 mL of solution. What is the molarity of a 10.0 mL sample of this solution?

2. A student dissolves 3.6 g of $C_6H_{12}O_6$ in enough H_2O to make 200 mL of solution. What is the molarity of a 10.0 mL sample of this solution?

3. A student dissolves 2.5 g of $CuSO_4{\cdot}5H_2O$ in enough H_2O to make 250 mL of solution. What is the molarity of a 10.0 mL sample of this solution?

4. A student dissolves 800 mg of NaOH in enough H_2O to make 125 mL of solution. What is the molarity of a 10.0 mL sample of this solution?

5. A student dissolves 5.0 g of $CuSO_4{\cdot}5H_2O$ in enough H_2O to make 25 mL of solution. What is the molarity of a 10.0 mL sample of this solution?

6. A student dissolves 180 mg of $C_6H_{12}O_6$ in enough H_2O to make 500 mL of solution. What is the molarity of a 10.0 mL sample of this solution?

Set 3 continued

7. A student dissolves 40.0 mg of NaOH in enough H_2O to make 50.0 mL of solution. What is the molarity of a 10.0 mL sample of this solution?

8. A student dissolves 750 mg of $CuSO_4{\cdot}5H_2O$ in enough H_2O to make 100 mL of solution. What is the molarity of a 10.0 mL sample of this solution?

9. A student dissolves 360 mg of $C_6H_{12}O_6$ in enough H_2O to make 250 mL of solution. What is the molarity of a 10.0 mL sample of this solution?

10. A student dissolves 40.0 mg of NaOH in enough H_2O to make 20.0 mL of solution. What is the molarity of a 10.0 mL sample of this solution?

<u>Chemistry Calculations: Mixtures of solutions without a reaction</u>

Use your mental math skills to answer these questions.

Set 4 - Perform these calculations *without a calculator*.

1. A 40.0 mL sample of 0.25 M $Ca(OH)_2$ is added to 60.0 mL of 0.50 M NaOH. What is the molar concentration of $OH^{1-}{}_{(aq)}$ in the resulting solution? (Assume the volumes are additive.)

2. A 40.0 mL sample of 0.50 M NaCl is added to 60.0 mL of 0.25 M $MgCl_2$. What is the molar concentration of $Cl^{1-}{}_{(aq)}$ in the resulting solution? (Assume the volumes are additive.)

3. A 40.0 mL sample of 0.125 M $Sr(OH)_2$ is added to 60.0 mL of 0.15 M KOH. What is the molar concentration of $OH^{1-}{}_{(aq)}$ in the resulting solution? (Assume the volumes are additive.)

4. A 50.0 mL sample of 0.30 M Na_3PO_4 is added to 50.0 mL of 0.60 M NaBr. What is the molar concentration of $Na^{1+}{}_{(aq)}$ in the resulting solution? (Assume the volumes are additive.)

5. A 50.0 mL sample of 0.25 M $Cu(NO_3)_2$ is added to 50.0 mL of 0.50 M $AgNO_3$. What is the molar concentration of $NO_3{}^{1-}{}_{(aq)}$ in the resulting solution? (Assume the volumes are additive.)

Set 4 continued

6. A 60.0 mL sample of 0.40 M $Mg(NO_3)_2$ is added to 40.0 mL of 0.60 M $MgSO_4$. What is the molar concentration of $Mg^{2+}{}_{(aq)}$ in the resulting solution? (Assume the volumes are additive.)

7. A 60.0 mL sample of 0.20 M $Fe(NO_3)_2$ is added to 40.0 mL of 0.25 M $LiNO_3$. What is the molar concentration of $NO_3{}^{1-}{}_{(aq)}$ in the resulting solution? (Assume the volumes are additive.)

8. A student adds 400 mL of distilled water to 200 mL of 0.15 M NaOH. What is the molar concentration of NaOH $_{(aq)}$ in the resulting solution?

9. A student adds 1.00 L of distilled water to 250 mL of 0.10 M HCl. What is the molar concentration of HCl $_{(aq)}$ in the resulting solution?

10. A student adds enough distilled water to 300 mL of 0.2 M NaOH to bring the final solution volume to 750 mL. What is the molar concentration of NaOH $_{(aq)}$ in the resulting solution?

<u>Chemistry Calculations: Mixtures of solutions with precipitation reactions</u>

Use your mental math skills to answer these questions. For each question, you should also write a chemical equation showing the products. Beneath the equation, construct a b-c-a table to identify the limiting reactant and amount of excess reactant remaining.

Set 5 - Perform these calculations *without a calculator*.

1. What is the final concentration of silver ions, $[Ag^{1+}]$, in solution when 100. mL of 0.050 M $NaCl_{(aq)}$ is mixed with 100. mL of 0.050 M $AgNO_3{}_{(aq)}$?

2. What is the final concentration of lead (II) ions, $[Pb^{2+}]$, in solution when 100. mL of 2.0 M $Pb(NO_3)_2{}_{(aq)}$ is mixed with 100. mL of 3.0 M KI $_{(aq)}$?

3. What is the final concentration of barium ions, $[Ba^{2+}]$, in solution when 10. mL of 8.0 M $BaCl_2{}_{(aq)}$ is mixed with 10. mL of 4.0 M $H_2SO_4{}_{(aq)}$?

4. What is the final concentration of calcium ions, $[Ca^{2+}]$, in solution when 200. mL of 0.150 M $Ca(OH)_2{}_{(aq)}$ is mixed with 100. mL of 0.150 M $Li_2CO_3{}_{(aq)}$?

5. What is the final concentration of copper (II) ions, $[Cu^{2+}]$, in solution when 100. mL of 0.4 M $KOH_{(aq)}$ is mixed with 100. mL of 0.5 M $Cu(ClO_3)_2{}_{(aq)}$?

Set 5 continued

6. What is the final concentration of iron (III) ions, [Fe^{3+}], in solution when 100. mL of 0.6 M $Fe(NO_3)_3$ *(aq)* is mixed with 50. mL of 1.2 M Cs_3PO_4 *(aq)*?

7. What is the final concentration of mercury (I) ions, [Hg_2^{2+}], in solution when 40. mL of 0.050 M $Hg_2(NO_3)_2$ *(aq)* is mixed with 20.0 mL of 0.050 M Na_2S *(aq)*?

8. What is the final concentration of strontium ions, [Sr^{2+}], in solution when 100. mL of 0.50 M $SrCl_2$ *(aq)* is mixed with 100. mL of 0.25 M K_2CrO_4 *(aq)*?

9. What is the final concentration of zinc ions, [Zn^{2+}], in solution when 100. mL of 0.80 M $ZnCl_2$ *(aq)* is mixed with 400. mL of 0.150 M LiOH *(aq)*?

10. What is the final concentration of calcium ions, [Ca^{2+}], in solution when 50.0 mL of 0.060 M $Ca(NO_3)_2$ *(aq)* is mixed with 25.0 mL of 0.060 M Na_2SO_4 *(aq)*?